

In Cartesian coordinates (see Fig. 1.3 and Appendix A.1), the force vector can be represented using the unit vectors $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$

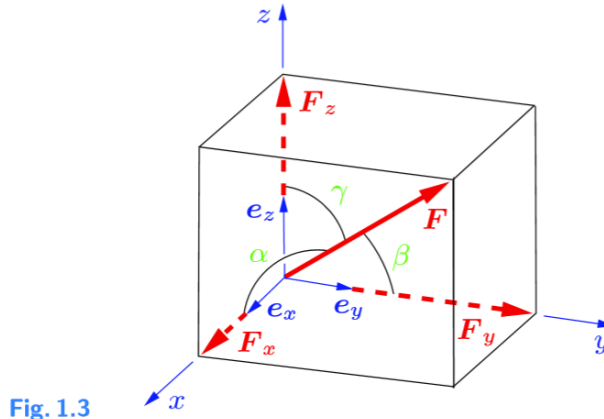


Fig. 1.3

by

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y + \mathbf{F}_z = F_x \mathbf{e}_x + F_y \mathbf{e}_y + F_z \mathbf{e}_z. \quad (1.1)$$

Applying Pythagoras' theorem in space, the force vector's magnitude F is given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}. \quad (1.2)$$

The direction angles and therefore the direction of the force follow from

$$\cos \alpha = \frac{F_x}{F}, \quad \cos \beta = \frac{F_y}{F}, \quad \cos \gamma = \frac{F_z}{F}. \quad (1.3)$$

— 2.5 Concurrent Systems of Forces in Space

It was shown in Section 2.2 that a force can unambiguously be resolved into two components in a plane. Analogously, a force can be resolved uniquely into three components in space. As indicated

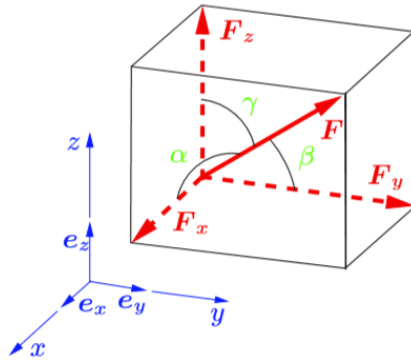


Fig. 2.17

in Section 1.2, a force $\mathbf{F}$ may be represented by

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y + \mathbf{F}_z = F_x \mathbf{e}_x + F_y \mathbf{e}_y + F_z \mathbf{e}_z \quad (2.12)$$

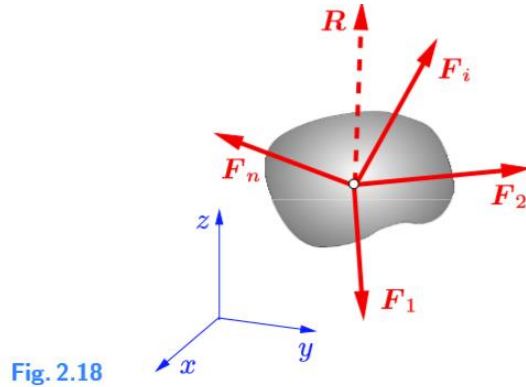
in a Cartesian coordinate system x, y, z (Fig. 2.17). The magnitude and the direction of $\mathbf{F}$ are given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2},$$

$$\cos \alpha = \frac{F_x}{F}, \quad \cos \beta = \frac{F_y}{F}, \quad \cos \gamma = \frac{F_z}{F}. \quad (2.13)$$

The angles α , β and γ are not independent of each other. If the first equation in (2.13) is squared and F_x , F_y and F_z are inserted according to the second equation the following relation is obtained:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1. \quad (2.14)$$



The resultant $\mathbf{R}$ of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ is obtained by constructing the parallelogram of the forces (see Section 2.1) which is expressed mathematically by the vector equation

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2. \quad (2.15)$$

In the case of a spatial system of n concurrent forces (Fig. 2.18), the resultant is found through a successive application of the parallelogram law of forces in space. As in the case of a system of coplanar forces, the resultant is the sum of the force vectors. Mathematically, this is written as (compare (2.2))

$$\mathbf{R} = \sum \mathbf{F}_i. \quad (2.16)$$

If the forces $\mathbf{F}_i$ are represented by their components F_{ix} , F_{iy} and F_{iz} according to (2.12), we obtain

$$\begin{aligned} \mathbf{R} &= R_x \mathbf{e}_x + R_y \mathbf{e}_y + R_z \mathbf{e}_z = \sum (\mathbf{F}_{ix} + \mathbf{F}_{iy} + \mathbf{F}_{iz}) \\ &= \sum (F_{ix} \mathbf{e}_x + F_{iy} \mathbf{e}_y + F_{iz} \mathbf{e}_z) \\ &= \left(\sum F_{ix} \right) \mathbf{e}_x + \left(\sum F_{iy} \right) \mathbf{e}_y + \left(\sum F_{iz} \right) \mathbf{e}_z. \end{aligned}$$

The coordinates of the resultant in space are thus given by

$$R_x = \sum F_{ix}, \quad R_y = \sum F_{iy}, \quad R_z = \sum F_{iz}. \quad (2.17)$$

The magnitude and direction of $\mathbf{R}$ follow as (compare (2.13))

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2},$$
$$\cos \alpha_R = \frac{R_x}{R}, \quad \cos \beta_R = \frac{R_y}{R}, \quad \cos \gamma_R = \frac{R_z}{R}. \quad (2.18)$$

A spatial system of concurrent forces is in *equilibrium* if the resultant is the zero vector (compare (2.10)):

$$\mathbf{R} = \sum \mathbf{F}_i = \mathbf{0}. \quad (2.19)$$

This vector equation is equivalent to the *three* scalar *equilibrium conditions*

$$\sum F_{ix} = 0, \quad \sum F_{iy} = 0, \quad \sum F_{iz} = 0, \quad (2.20)$$

which represent a system of three equations for three unknowns.