

2.4 Examples of Coplanar Systems of Forces

To be able to apply the above theory to specific problems, a few idealisations of simple structural elements must be introduced. A structural element whose length is large compared to its cross-sectional dimensions and that can sustain only tensile forces in the direction of its axis, is called a *cable* or a *rope* (Fig. 2.11a). Usually the weight of the cable may be neglected in comparison to the force acting in the cable.

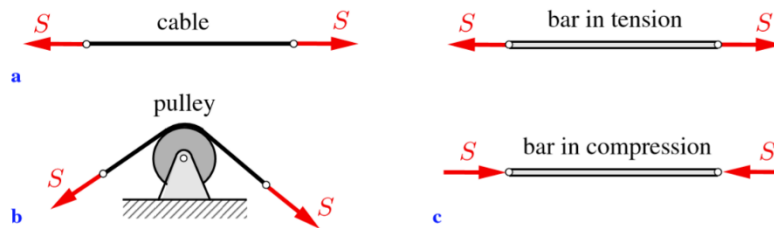


Fig. 2.11

As explained in Section 1.5, the force acting at the point of contact between two bodies can be made visible by separating the bodies (Fig. 2.12a, b). According to Newton's third law (action = reaction) the contact force K acts with the same magnitude and in an opposite direction on the respective bodies (Fig. 2.12b). It may be resolved into two components, namely, the *normal force* N and the *tangential force* T , respectively. The normal force is perpendicular to the plane of contact, whereas the tangential force lies in this plane. If the two bodies are merely touching each other (i.e., if no connecting elements exist) they can only be pressed against each other (pulling is not possible). Hence, the normal force is oriented towards the interior of the respective body. The tangential force is due to an existing roughness of the surfaces of

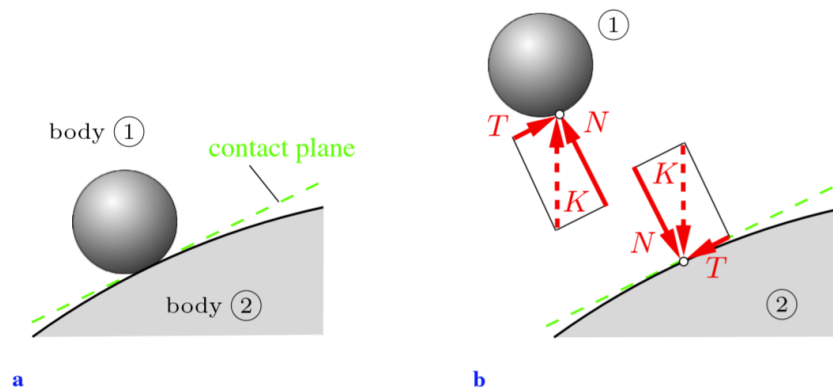
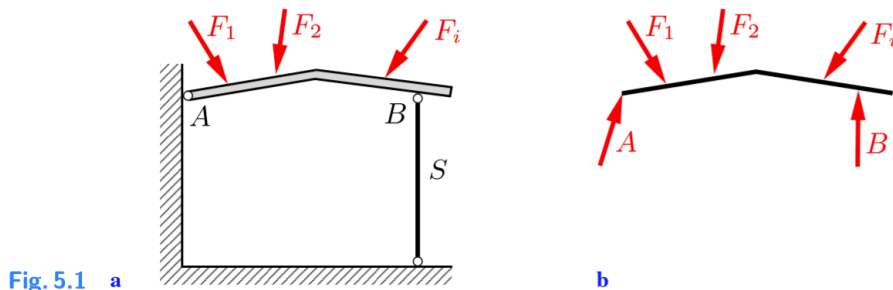


Fig. 2.12

the bodies. In the case of a completely smooth surface of one of the bodies (= idealisation), the tangential force T vanishes. The contact force then coincides with the normal force N .

5.1.1 Supports

Structures can be classified according to their geometrical shape and the loads acting on them. A straight slender structural element (cross-sectional dimensions much smaller than its length) that is loaded solely in the axial direction (tension or compression) is called a *bar* or a *rod* (see Section 2.4). If the same geometrical object is subjected to a load perpendicular to its axis, it is called a *beam*. A curved beam is usually designated as an *arch*. Structures consisting of inclined, rigidly joined beams are called *frames*. A plane structure with a thickness much smaller than its characteristic in-plane length is called a *disk* if it is solely loaded in its plane, e.g., by in-plane forces. If the same geometrical structure is loaded perpendicularly to its midplane it is called a *plate*. If such a structure is curved it is a *shell*.



Structures are connected to their surroundings by *supports* whose main purpose is to fix the structure in space in a specific position. Secondly, supports transmit forces. As a simple example, consider the “roof” in Fig. 5.1a, loaded by external forces F_i , joined at A to a vertical wall by a pin, and supported at B by the strut S . Forces are transmitted to the wall and the ground via the supports A and B . According to the law of action and reaction (actio = reactio) the same forces are exerted in opposite directions from the wall and the ground onto the roof. These forces from the environment onto the structure are reaction forces (cf. Section 1.4), and are termed *support reactions*. They become visible in

the free-body diagram (Fig. 5.1b), where they are generally denoted by the same symbols as the supports, i.e. by A and B in this example.

The following discussion is limited to single-part structures located and loaded in a plane. A free body with no restraints has three degrees of freedom, i.e., it can be independently displaced in three different ways: by two translations in different directions and by one rotation about an axis perpendicular to the plane (cf. Section 3.1.4). Supports (restraints) reduce the feasible displacements: each support reaction imposes a constraint. Let r be the number of support reactions. Then the number f of degrees of freedom of a body in a plane is given by

$$f = 3 - r \quad (5.1)$$

(for exceptions see Section 5.1.2).

We now will consider different types of supports and classify them by the number of support reactions involved.

Supports that can transmit only one single reaction ($r = 1$). Examples of this type of support are the roller support, the simple support and the support by a strut, cf. Fig. 5.2a–c. In this case, the direction of the reaction force is known (here vertical), whereas its magnitude is unknown.

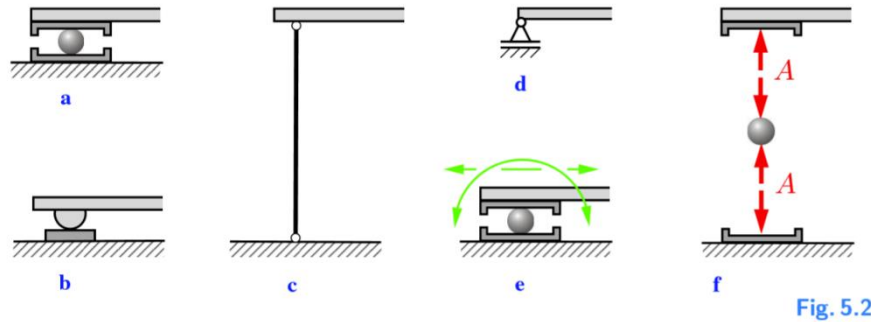


Fig. 5.2

Figure 5.2f shows the free-body diagram for the roller support. If the contact areas are assumed as frictionless, all contact forces can be considered to act perpendicular to the respective contact surfaces. With this assumption the action line of the reaction force A is determined. Figure 5.2e indicates the displacements that are unconstrained by the support: a horizontal translation and a rotation. A vertical translation is excluded through the support's restraint. If the support reaction A changes its sign, i.e., if it is reversed in the direction along the action line, a lift-off must be prevented by an appropriate support construction. From now on a simple support will be depicted by the symbol shown in Fig. 5.2d.

Supports that can transmit two reactions ($r = 2$). Examples of this type of support are the hinged support and the support by two struts (Fig. 5.3a, b), which are depicted symbolically in Fig. 5.3c.

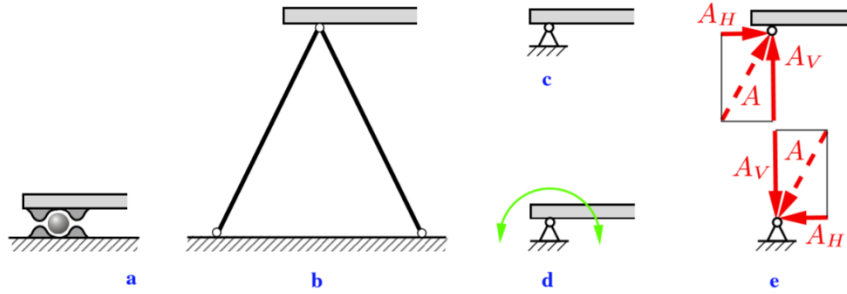


Fig. 5.3

As shown in Fig. 5.3d, the hinged support allows a rotation but not a displacement in any direction. Accordingly, it can transmit a reaction force A of arbitrary magnitude and arbitrary direction that can be resolved into its horizontal and vertical components A_H and A_V (Fig. 5.3e).

Additional variants of supports transmitting two reactions are the parallel motion and the sliding sleeve (Fig. 5.4a, b). Their free-body diagrams (Fig. 5.4c, d) show that in both cases one force and

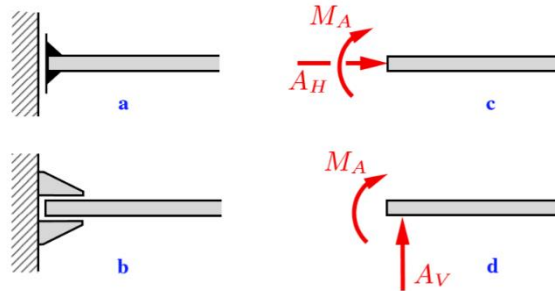
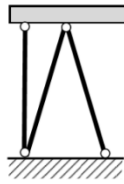


Fig. 5.4

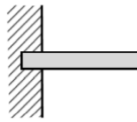
one couple moment can be transmitted. A displacement in one single direction is possible in each case; a displacement in any other direction or a rotation are not possible.

The rotational degree of freedom disappears if a support by two struts is complemented by an additional, somewhat shifted, third strut (Fig. 5.5a): the structure becomes immobile. In addition to the two force components, the support can now also transmit a couple moment, i.e., in total *three reactions*: $r = 3$.

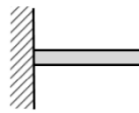
The same situation appears in the case of a clamped support (fixed support) according to Fig. 5.5b which symbolically is depicted in Fig. 5.5c. The free-body diagram in Fig. 5.5d shows that the clamped support can transmit a reaction force A of arbitrary magnitude and direction (or A_H and A_V) and a couple moment M_A .



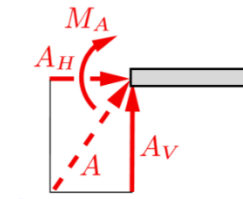
a



b



c



d

Fig. 5.5